Engagement, Disengagement and Exit

(EXTREMELY PRELIMINARY, INCOMPLETE & EXPLORATORY)

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Abstract

This paper considers the possibility of information transmission across groups when any group may choose to unilaterally exit society. My main concern is the tradeoff social groups face between the informational benefits of associating with other groups in a large society versus the costs imposed by preference diversity on receiving their preferred outcome. When these costs are sufficiently high, groups may prefer exit to association. The results of this paper characterize the associations of groups that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. The results demonstrate that there can be benefits or costs associated with the inclusion of preference extremists in a diverse society, whether or not those groups choose to actively communicate with outgroups. The results also speak to both institutional and intra-group mechanisms for fostering communication across groups.

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1 Introduction

Democratic theorists have long recognized the value of information to a well-functioning society. Information is required for a population to critically evaluate its leaders, to confront conflicts that arise, and, more generally, to secure its well-being through its own agency. In this vein, John Adams writes that "...wherever a general knowledge and sensibility have prevailed among the people, arbitrary government and every kind of oppression have lessened and disappeared in proportion."[1] This view suggests that information aids democracy because it enables individuals to reflect upon their own situations and empowers them to take action; it serves as a check against leaders pursuing undemocratic goals. A different view of the role of information in democratic society is that information *per se* constitutes the source of a democracy's value. In documenting the history of Athenian governance, Ober argues that the participatory nature of Athenian democracy served to consolidate information that was widely dispersed throughout the population, and that this aggregation and distribution of knowledge played a causal role in Athens' success relative to its peer polities. The idea that democratic institutions serve as systems for "...organizing what is known by many disparate people"[2] has arisen in many manifestations, in both ancient and modern work.[3]

Regardless of the view that one takes concerning the role of information in a democratic society, it is clear that when individuals hold private information the question of whether or not they choose to share that information with others becomes a strategic consideration. A large literature has been devoted to the study of strategic information transmission, with a key and robust finding being that as the preferences of individuals become increasingly divergent, their ability to credibly communicate with one another is reduced. This phenomenon is exacerbated in settings in which communication between individuals is both costless and non-verifiable.[4] However, it is precisely

[1] Adams (1765).

[2] Ober (2008, p. 2).

[3] Arguments drawing upon the notion of "wisdom of crowds" can be found in work by Condorcet (1785), Hayek (1945), Surowiecki (2005) and Page (2008), among many others.

[4] Crawford and Sobel (1982).

this kind of setting that might characterize a democratic society in which information is highly decentralized, there are no barriers to whom one may associate and communicate with, and people may freely say what they like without fear of punishment.

I begin this paper by presuming that information is valuable for decision making, and by acknowledging the fact that diversity, free association and free speech are the hallmarks of many democratic societies. Phrased differently, much of communication in a free society is cheap talk. Using this observation as a starting point, I am interested in the possibility of information transmission across diverse social groups that are internally homogenous, and when it is (or is not) in the interest of a group to co-exist with other groups. My main concern is the tradeoff that groups face between the informational benefits of associating with others in a large society versus the policy costs of that association; if, for example, several distinct groups choose to associate with each other, each must bear a negative externality stemming from their divergent preferences over outcomes. If these costs are sufficiently high, one or more groups may prefer unilateral exit to association.[5]

The results of this paper characterize the associations of groups that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. These "associations" are precisely the collections of groups for which some information transmission between groups is possible, and for which the benefits of this information outweigh the costs stemming from preference diversity within the association. In equilibrium the choice of a group to "associate" necessarily represents an improvement in expected utility relative to the choice to "exit." As a group can always choose exit, members of an equilibrium association are always made weakly better off by their decision to associate. Consequently, while larger associations may not maximize social welfare in a utilitarian sense, in equilibrium they virtually always improve the well-being of some group relative to its exit option while maintaining the well-being of all other members of the association above minimum levels.

[5]Several recent papers address questions concerning group interactions in heterogeneous societies. See, for example, Eguia (2012) (assimilation) and Schnakenberg (2013) (identity and symbolic behavior).

Thus, while not precisely Rawlsian, larger associations can be considered normatively desirable in this sense.

To briefly and informally describe the dynamics underlying the model, suppose that there are two people, each unaware of a true "state of nature." If known, this state of nature would perfectly inform individual behavior. To be more specific, suppose that each individual cares about both music and math education, and each needs to choose a division of their child's time to dedicate to each subject. For each person there is an optimal division of time to devote to these two activities. This optimal amount depends on both the true effect of the division on a child's well-being and on the parent's own biases concerning the relative importance of music versus math education. While no one knows the true effect of music versus math on childhood development, each person receives a signal that can be used to inform his or her choice. And while each individual makes a personal choice about their own child's activities, these choices affect everyone, because each person sees the choices of others and has preferences concerning the societal "music versus math" divide.

Suppose that these two people can costlessly communicate with each other about whether music or math is better. If communication is truthful and credible then it will enable both people to see a larger collection of signals and thus arrive at a more accurate estimate of the true state of nature prior to making the costly decision concerning their child's division of time. Of course, the tradeoff the people face is that with this larger pool of information comes a larger set of potential biases informing the decisions of others; a person may like having better information about the state of nature, but may dislike how the other person chooses to utilize his or her own improved information. If this tradeoff becomes too great, the individuals may choose to part ways; they may choose to forego each other's information in favor of no longer being privy to the choices made by the other person.

The model I consider is not as much concerned with the behavior of individuals as with the behavior of groups, which may have information that varies in quality. But in keeping with the story just told, a group may choose to forego the informational benefits of communication with outgroups in favor of being more able to target outcomes to its own biases. In this instance, the group chooses *exit*. In the "music versus math" example, a music-loving group's choice of exit could be reflected in the group's decision to establish its own music-intensive school. Alternatively, the group may choose to engage in a process of communication with outgroups but not reveal any meaningful information to those groups. This could occur, for example, in a situation in which the music-loving group exists in a society full of math whizzes. In this case, if the group receives private information that perhaps the optimal tradeoff between music and math favors math more than previously thought, it might choose to withhold that information from outgroups to prevent them from biasing outcomes even farther toward math. At the same time, associating with the math whizzes may still be optimal for the music lovers as it enables them to reap the informational benefits of the (truthful) messages of others. I refer to a group's choice to associate with others but conceal its information from them as *disengagement*. Last, the group may choose to associate with others and to truthfully reveal its information to all. Such groups benefit from truth-telling because their honesty better informs the choices of others, and the choices of others directly affect everyone in the association. I refer to these groups as *engaged.*

The paper proceeds as follows. In Section 2 I present the model of group association and cheap talk, the latter of which which utilizes a messaging technology similar to one developed by Galeotti et al. (2013).[6] Section 3 presents several general results, including a characterization of the conditions required for truthful messaging and for voluntary association, a partial ranking of equilibria on the basis of social welfare, and a more detailed discussion of the two-group case. Section 4 works through two examples to provide intuition for the kinds of comparative statics that emerge from the model. Both examples focus on how equilibria vary as the bias of a third group changes from centrist to extreme. Section 5 discusses how allocating policy discretion across

[6] In terms of the messaging subgame I consider, the main distinction between the setting I consider and theirs is that the agents in my model can be differentiated in part by the number of signals they receive, whereas agents in their model receive a single signal. A number of recent works have employed this framework to address various topics in political economy including Dewan and Squintani (2012) (endogenous factions); Dewan et al. (2011) (optimal executive structure); Patty (2013) (inclusion and exclusion of agents); Patty and Penn (2012) (sequential decision-making); and Gailmard and Patty (2013) (delegation of authority).

a collection of groups can be used as an instrument to induce better communication and grow the size of an association. Section 6 discusses how, by foregoing some of its own information, a group can improve its ability to communicate with another group. This strategy is compared with a different mechanism for improving communication, namely obfuscation of a group's own information in the form of playing a semi-separating strategy. Section 7 presents a different intra-group mechanism for improving communication with an outgroup: disaggregation of the group's own information by limiting within-group communication. Section 8 concludes.

2 The Model

I consider a society consisting of a collection of groups $g \in G$, with $|G| = n$, and with each group consisting of n_g individuals ("members of g"). Every person belongs to a single group. Groups are differentiated solely on the basis of their respective sizes and the preferences of their members. These preferences are represented by a vector of biases, $\beta = (\beta_1, ..., \beta_n)$, which are common knowledge, as are the number of individuals in each group, n_g. An individual's bias will affect his payoff from both his own activities and the activities undertaken by the individuals he has chosen to associate with. Every individual undertakes an activity $y \in \mathbb{R}$, and a member of g's payoff to a particular activity y is:

$$u_g(y, \theta; \beta_g) = -(y - \theta - \beta_g)^2,$$

where $\theta \in \Theta$ represents a "state of the world" drawn from a Uniform$[0,1]$ distribution over state space $\Theta = [0,1]$. Upon realization of θ, every person in the population receives a conditionally independent, private signal $s_i \in \{0,1\}$, according to the probability mass function:

$$\Pr[s_i = x|\theta] = \begin{cases} 1-\theta & \text{if } x = 0 \\ \theta & \text{if } x = 1. \end{cases} \tag{1}$$

Let $s_g = \sum_{i \in g} s_i$ be the number of positive signals received by the members of group g. Thus, a member of g considers an "ideal action" to be $\theta + \beta_g$, and prefers activities to be as close to this action as possible.

2.1 Action and communication

For reasons that will become clear later, I begin by presuming that group members will truthfully communicate their private signals, s_i, to one another so that I may conceive of "the group" as a unitary actor that will receive n_g total signals. This is because I am interested in the tradeoffs groups face when choosing whether to exit or enter society at large, and how both the size of a group and its relative bias affect this decision.[7] While I assume that individuals will always communicate truthfully with members of their own group, groups may choose to associate with "society at large" *prior to realizing their own signals*. A choice to associate with the larger society implies that the group agrees to both participate in a process of public communication with all other groups that have similarly chosen to associate with society at large, and to receive payoffs that are dependent on the actions of the groups in the association.

As noted above, each group faces a tradeoff when choosing whether or not to associate with society at large. On the one hand, members of the group could choose to disassociate from the whole, and to receive a payoff that is dependent on only the activities of the group members and the information that that subset of individuals could provide. On the other, if the group chooses to associate with society at large then members can, potentially, reap the informational benefits that the larger pool of signals may afford. However, association comes at the cost of incurring a payoff that dependent on the actions of all members of the association. Thus, each member of an association incurs a negative externality associated with the actions of others who have similarly chosen to associate, but who hold different biases.

Leaving aside the role of communication for the moment, social outcomes are captured via a decision that every group g makes over its choice of an activity, $y_g \in \mathbb{R}$ with $y = (y_1, ..., y_n)$. If a group chooses association its members receive a payoff that is dependent on the activities of all

[7] Clearly, larger groups will have better information about θ than smaller groups, and a later section of this paper considers whether members of a large group can be made better off by committing to limit their own within-group communication, as doing so enables groups to credibly commit to having less information about θ. As I will discuss later, this commitment may induce other groups to regard information communicated by that group as truthful.

members of the association. This payoff is

$$u_g^{\mathtt{a}}(y, R|\theta) = -\sum_{h\in R} \alpha_h^R (y_h - \theta - \beta_g)^2, \tag{2}$$

with α_h^R being an exogenous weight attached to group h's actions by each member of association R. For any $R \subseteq G$, α_h^R captures the relative influence of h's decision within the association. These α terms could, for example, be proportional to the population of each group within the association, proportional the wealth of the groups, or something else entirely. If a group chooses exit then its payoff is solely dependent on its own choice of activity, y_g so that

$$u_g^{\mathtt{x}}(y|\theta) = -(y_g - \theta - \beta_g)^2. \tag{3}$$

The above payoffs to association and exit reflect the fact that, with equal information about state of nature θ, a group would always choose exit over association, as exit enables the group to perfectly target its activity to its bias and avoid the externalities associated with preference diversity. However, a group that chooses exit cannot receive information from any other group, and so updates its estimate of θ solely on the basis of the number of positive signals received by its members, s_g. Conversely, members of an association may receive information from the groups that have similarly chosen association, in the form of a public, cheap talk message that each group sends to the association concerning the number of positive signals its members have received. If informative, these messages will improve each association member's estimate of θ. Thus, groups choosing association also choose a message to be conveyed to the other groups that have chosen association. To summarize, groups make two types of choices, an association decision $a_g \in \{\mathtt{a}, \mathtt{x}\}$ denoting association or exit, respectively, and, if choosing association, a message to be sent to the other groups in the association, $m_g \in \{0, ..., n_g\}$, communicating the sum of positive signals claimed to have been received by the group.

2.2 Messaging Equilibria and Societal Stability

I am interested in characterizing the types of associations that can form, and group behavior within these associations, when communication and association decisions are strategic. My focus is on pure strategy perfect Bayesian Nash equilibria. Groups' actions are decomposed into two parts, which consist of an association decision and a messaging strategy. A (pure) association decision is simply a choice $a_g \in \{\mathtt{a}, \mathtt{x}\}$. If $a_g = \mathtt{a}$, a messaging strategy ρ_g maps a sum of signals received by group g and a set of associators, $R \subseteq G$, into a public message to the association, m_g. Thus, $\rho_g : \{0, ..., n_g\} \times 2^{|G|} \to \{0, 1, 2, ..., n_g\}$.

The signaling technology defined in Equation 1 implies that a player's posterior belief about θ after observing m trials (signals) and k successes (observations of $s = 1$) is characterized by a Beta$(k + 1, m - k + 1)$ distribution, which implies:

$$\begin{aligned} E(\theta|k, m) &= \frac{k+1}{m+2}, \text{ and} \\ V(\theta|k, m) &= \frac{(k+1)(m-k+1)}{(m+2)^2(m+3)}. \end{aligned} \tag{4}$$

It follows that if group g observes the truthful revelation of k successes and $m - k$ failures, then it is always optimal for g to select:

$$y_g^*(k, m) = \frac{k+1}{m+2} + \beta_g. \tag{5}$$

For the sake of parsimony, and to clarify the arguments I wish to make, I focus on communication strategies within the messaging subgame for the association that are either separating or pooling.[8] This focus simplifies the analysis by enabling us to consider only three actions taken by groups in equilibrium, with these actions implicitly capturing the group's association decision, messaging strategy and policy choice. Groups can either exit, associate and communicate truthfully, or associate and babble. The equilibria defined below can therefore be characterized by

[8]The focus on communication strategies that can take one of two forms (truthful or uninformative) reduces to a focus on pure strategies when players have only two signals. Thus, while analogous to the analysis in Galeotti et al., my focus on separating equilibria is a stronger restriction than pure strategies are in their framework. The additional restrictiveness stems from the larger message space considered here. As those authors note, the existence of mixed strategies in their setting is possible, as it is here. Moreover, this model also yields semi-separating pure strategy equilibria, which are discussed in more detail through an example I construct in Section 6. At the same time, the focus on separating equilibria greatly simplifies the analysis while capturing qualitative features of the model that would similarly be found by expanding my scope to consider these different types of equilibria.

considering divisions of G into three possibly empty and mutually disjoint sets: E, D, and X, where $E \cup D \cup X = G$. These sets correspond, respectively, to the groups that choose $a_g = \texttt{a}$ and $\rho_g(s_g, R) = s_g$ for all $s_g \in \{0, ..., n_g\}$; the groups that choose $a_g = \texttt{a}$ and $\rho_g(s_g, R) = 0$ for all $s_g \in \{0, ..., n_g\}$; and the groups that choose $a_g = \texttt{x}$. I denote the collection of E, D, X divisions of G by $\mathcal{S}$ with element ("society") $\sigma \in \mathcal{S}$.

I refer to groups in E as those that have chosen to engage, as these groups will both associate with society at large and truthfully reveal their information to the other groups that have chosen association. I refer to groups in D as those that have chosen to disengage, as these groups will associate with society at large but reveal no information to other groups. Groups in X have chosen to exit society, receiving no information from outgroups and taking actions informed solely by the information provided by their own members. The set of groups $R = E \cup D$ is termed the *association*, as these groups have chosen to engage in a public messaging game. Let $n_E = \sum_{g \in E} n_g$ and $s_E = \sum_{g \in E} s_g$ represent, respectively, the number of individual group members in E and the sum of positive signals received by those members. Sequential rationality implies that actions, y_g, maximize groups' expected payoffs given their own signals and the messages they receive. Thus, for a division of groups $\sigma \in \mathcal{S}$ with $\sigma = \{E, D, X\}$, let $y_\sigma = (y_{g,\sigma})_{g \in G}$ denote a sequentially rational profile of actions for society σ, so that:

- For $g \in E$, $y_{g,\sigma} = y_g^*(s_E, n_E)$,
- For $g \in D$, $y_{g,\sigma} = y_g^*(s_E + s_g, n_E + n_g)$,
- For $g \in X$, $y_{g,\sigma} = y_g^*(s_g, n_g)$.

Before defining the notion of societal stability used for the remainder of the paper, I define a set of *messaging equilibria* for each possible association $R \subseteq G$ that satisfy the equilibrium refinements discussed above.

Definition 1 *For any $R \subseteq G$, a division of R into two disjoint subsets, $\{E, D\}$ with $E \cup D = R$, is a **messaging equilibrium** for R if the following three conditions are met:*

1. Individuals have equilibrium beliefs. For all groups in association R, public messages sent by groups $g \in E$ are taken as equal to s_g and public messages sent by groups in $g \in D$ are disregarded as uninformative.

2. Actions are sequentially rational given groups' own signals and the messages they receive.

3. Groups that truthfully message have no incentive to lie. This means that for all $g \in E$, messaging strategy $\rho_g(s_g, R) = s_g$ for all $s_g \in \{0, 1, ..., n_g\}$ offers g a weakly higher expected payoff than any other strategy that could be taken by g, given the (correct) beliefs, actions and equilibrium messaging strategies of the other groups in association R.

Using the above definition of a messaging equilibrium for R, let

$$\mu(R) = \{\{E, D\} : \{E, D\} \text{ is a messaging equilibrium for } R\}.$$

Thus, $\mu(R)$ is the set of all messaging equilibria for association R. Last, let $EU_g(y_\sigma)$ be the expected utility of group g after messaging, association and (implicit) activity choices have occurred as dictated by σ. We are now in a position to define a notion of societal stability.

Definition 2 *An $\sigma \in \mathcal{S}$ with $\sigma = \{E, D, X\}$ is **stable** if the following three conditions are met:*

1. The set $\{E, D\}$ constitutes a messaging equilibrium for $R = E \cup D$. Thus, $\{E, D\} \in \mu(E \cup D)$.

2. For $g \in X$ actions are sequentially rational given the groups' own signals.

3. Groups have no desire to change their association decisions.

   - For all $g \in R$, $EU_g(y_\sigma) \geq EU_g(y_{\sigma'})$, where $\sigma' = \{E \setminus \{g\}, D \setminus \{g\}, X \cup \{g\}\}$.

   - For all $g \in X$, $EU_g(y_\sigma) \geq \max_{\{E', D'\} \in \mu(R \cup \{g\})} EU_g(y_{\sigma'})$, where $\sigma' = \{E', D', G \setminus E' \cup D'\}$.

The final stability condition deserves particular attention. This condition states that no group can strictly benefit by changing its association decision. For groups in association R the condition is straightforward, because it implies that they prefer remaining in R, given the current messaging equilibrium $\{S, E\}$, to exit. For groups in X the association decision poses a potential ambiguity, because if $g \in X$ chooses to enter association R the messaging strategies of the groups in R may change. Thus, a group g that has chosen exit must compare the expected utility of exit to the expected utility of being in association $R \cup \{g\}$ *given a new messaging equilibrium that his entry will generate*. As $\mu(R \cup \{g\})$ may not be single-valued, this calculation involves g choosing an element of $\mu(R \cup \{g\})$ to evaluate potential benefits of entry with respect to. In this case I assume that g makes its calculation using a messaging equilibrium associated with association $R \cup \{g\}$ that maximizes the expected utility of association for g. While this assumption seems strong, I discuss in Section 3.1 that if a messaging equilibrium for association R maximizes the expected utility of some $g \in R$ then it maximizes the expected utility of *every* $g \in R$; in other words, groups in an association have the same preferences over messaging equilibria. This stems from the fact that the only gain to association is a reduction in the residual variance of each group's estimate of θ due to information transmission. Due to the "shared policy-making " nature of group utility functions, each group in the association benefits equally from this reduction in variance. Thus, g evaluates its entry into R by considering an equilibrium that is utility-maximizing for all members of R, including itself.

3 General Results

I begin this section by deriving conditions for truthful communication by group g to association R, and by deriving the conditions that characterize groups' association decisions. Together, the conditions can be used to characterize instances of societal stability. The second part of this section presents several welfare implications of the model and the final part of this section considers these conditions for the special case of a society composed of two groups. I begin with the following lemma, which shows that the incentive for a group to truthfully communicate to an association R is most difficult to satisfy when that group seeks to misstate the signal of a single one of its members. Thus, the lemma shows that if it is profitable for g to misstate the signals of some of its members then it is profitable for g to misstate the signal of a single one of its members. The lemma is used to simplify the condition for truthful communication by enabling us to only consider instances in which it is profitable to misstate a single signal. All proofs are in the Appendix.

Lemma 1 *Let g have n_g members who have received a total of $s_g \leq n_g$ positive signals. If revealing $\tilde{s} \neq s_g$ positive signals represents a profitable lie for g, then one of two conditions holds. If $\tilde{s} > s_g$ then claiming $s_g + 1$ positive signals is also a profitable lie. If $\tilde{s} < s_g$ then claiming $s_g - 1$ positive signals is also a profitable lie.*

Lemma 1 states that if Group g has an incentive to communicate dishonestly then g also has an incentive to over- or under-report s_g by a single signal. This result enables us to derive the following condition for truthful public communication from Group g to association R. Of course, the condition must take into account the fact that some groups in R will themselves communicate truthfully while others will not. The existence of both of these types of groups within the association affects the incentives for g to communicate truthfully. On the one hand, the existence of babblers within R means that fewer credible signals are revealed to others within the association. This increases the "manipulative impact" of a false signal by g, and thus lowers g's incentive to reveal a false signal by potentially increasing the cost of such a signal. On the other hand, a false signal by g has a different, lower, manipulative impact on the policy choice of a babbling group than it does on the policy choice of a truthful group. This is because groups that babble have strictly more information than groups that are truthful: they have their own, private, information in addition to the information provided by the groups in E. For the condition below, let $E_{-g} = E \setminus \{g\}$. In the same fashion, let $s_{E_{-g}} = \sum_{h \in E_{-g}} s_h$, the sum of positive signals received by the groups in E_{-g}, and let $n_{E_{-g}}$ be the total number of signals received by the groups in E_{-g}.

Condition 1 *Let R be an association, with $E \subseteq R$ being the groups that truthfully reveal their signals and $D \subseteq R$ being those that babble. The* ***truthful messaging condition*** *for group $g \in E$ is then:*

$$\sum_{h \in E_{-g}} \frac{\alpha_h^R}{2(n_{E_{-g}} + n_g + 2)^2} + \sum_{j \in D} \frac{\alpha_j^R}{2(n_{E_{-g}} + n_j + n_g + 2)^2} \geq \Big| \sum_{h \in E_{-g}} (\frac{\alpha_h^R}{n_{E_{-g}} + n_g + 2})(\beta_h - \beta_g) + \sum_{j \in D} (\frac{\alpha_j^R}{n_{E_{-g}} + n_j + n_g + 2})(\beta_j - \beta_g) \Big|.$$

The truthful messaging condition only characterizes one aspect of a group's strategy: whether it is able to credibly communicate with association R. I now move on to a condition that I term *voluntary association.* This condition characterizes the requirement that each group in association R would prefer remaining in R to exit. Let $V(\theta|m)$ be the expected variance of a posterior belief about θ after receiving m signals. The assumption of a quadratic utility function, combined with sequential rationality, enables us to express the equilibrium expected utility to player g from action profile $\sigma = \{E, D, X\}$ as the following:

$$EU_g(y_\sigma) = -V(\theta|n_g)$$

for $g \in X$, and[9]

$$EU_g(y_\sigma) = -\sum_{k \in R} \alpha_k^R (\beta_k - \beta_g)^2 - \sum_{h \in E} \alpha_h^R V(\theta|n_E) - \sum_{j \in D} \alpha_j^R V(\theta|n_E + n_j) \tag{6}$$

for $g \in R = E \cup D$.

The voluntary association condition requires that for each $g \in R$, the expected utility from remaining in R exceeds the expected utility g would receive from forgoing the association in favor of implementing its own policy. The condition also requires that each $h \in X$ faces no possible

[9]Recall that a group choosing exit can perfectly target its choice of policy y_g to its bias, β_g, and so the group's expected utility is only a function of the variance surrounding its own decision.

benefit from joining the association. This requires that for each group that has chosen exit from some association R, there is no messaging equilibrium for association $R' = R \cup \{h\}$ that yields h strictly higher expected utility than would be received by remaining out of the association. For the statement below, let $R' = R \cup \{h\}$. We can now express the voluntary association condition as follows.

Condition 2 *The* ***voluntary association condition for group*** $\mathrm{g} \in \mathbf{R}$ *requires that:*

$$-\sum_{k\in R}\alpha_k^R(\beta_k-\beta_g)^2-\sum_{h\in E}\alpha_h^R V(\theta|n_E)-\sum_{j\in D}\alpha_j^R V(\theta|n_E+n_j) > -V(\theta|n_g).$$

The ***voluntary association condition for group*** $\mathrm{h} \in \mathbf{X}$ *requires that:*

$$-V(\theta|n_h) \geq \max_{\{E',D'\}\in\mu(R')} -\sum_{k\in R'}\alpha_k^{R'}(\beta_k-\beta_h)^2-\sum_{h\in E'}\alpha_h^{R'} V(\theta|n_{E'})-\sum_{j\in D'}\alpha_j^{R'} V(\theta|n_{E'}+n_j).$$

One immediate implication of Condition 2 is that, if $\sum_g \alpha_g^R \geq 1$ for all associations R, there can never be a stable configuration of groups in which the set of engagers, E, is empty but the association is nonempty. If this occurs, then the group with the most information can always benefit from exit; if it exits, this group can both make a more informed decision than the other groups and perfectly target policy to its own bias. This is stated in the following corollary.

Corollary 1 *Let $\sum_g \alpha_g^R \geq 1$ for all associations R. If $\sigma = \{E, D, X\}$ is such that $E = \emptyset$ and $D \neq \emptyset$, then σ cannot be stable and, in particular, will violate voluntary association for some group in $g \in D$.*

3.1 Social welfare

For a given collection of groups, Conditions 1 and 2 can be used to completely characterize the set of stable societal configurations of groups into sets E, D and X. There always exists at least one stable configuration of groups and this is $\sigma = \{\{\}, \{\}, G\}$, in which every group has chosen exit. In this case the association is empty, and so a group choosing “association” will simply be associating with itself. As the group is indifferent between entry and exit, by Definition 2 exit is a stable choice. Oftentimes, however, there are multiple stable configurations of groups. Galeotti et al. demonstrate that when multiple equilibria exist in their framework they can be Pareto-ranked and characterized by a straightforward rule. The spirit of their result translates in part to the setting considered here. For a fixed association R the set of messaging equilibria $\mu(R)$ can similarly be Pareto-ranked, although the rule that characterizes the ranking in Galeotti et al. no longer holds here because of the varying weights α attached to the policy decisions of the groups. However, when multiple equilibria exist that correspond to *different* stable associations, these equilibria can oftentimes not be Pareto ranked. This stems from the nature of the association decision, and the fact that any group choosing exit obtains the minimum level of utility they could be expected to receive. Therefore, if R and R' are different associations that correspond to stable equilibria σ and σ' respectively, and if $g \in R \setminus R'$ and $j \in R' \setminus R$, then σ and σ' cannot be Pareto ranked: g prefers σ to σ' and j prefers σ' to σ.

As shown earlier, g's ex ante expected utility to association with R is

$$EU_g(y_\sigma) = -\sum_{k\in R} \alpha_k^R(\beta_k - \beta_g)^2 - \sum_{h\in E} \alpha_h^R V(\theta|n_E) - \sum_{j\in D} \alpha_j^R V(\theta|n_E + n_j).$$

The first term in this equation is constant for a given g and R while the next two terms are dependent on the messaging strategies of the members of R and are the same for all members of R. Thus, messaging equilibria $\mu(R) = \{\{E_1, D_1\}, \{E_2, D_2\}, ...\}$ are Pareto ranked on the basis of maximization of

$$-\sum_{h\in E} \alpha_h^R V(\theta|n_E) - \sum_{j\in D} \alpha_j^R V(\theta|n_E + n_j).$$

The term $V(\theta|m)$ reduces to the following fraction, leading to the following result:

$$\int_{\theta=0}^{1} \sum_{k=0}^{m} \binom{m}{k} \theta^k (1-\theta)^{(m-k)} \left(\frac{(k+1)(m-k+1)}{(m+2)^2(m+3)}\right) = \frac{1}{6(m+2)}.$$

Proposition 1 *For a fixed association R, messaging equilibrium $\{E, D\}$ Pareto dominates $\{E', D'\}$*

if and only if

$$-\sum_{h\in E}\frac{\alpha_h^R}{6(n_E+2)}-\sum_{j\in D}\frac{\alpha_j^R}{6(n_E+n_j+2)}>-\sum_{h\in E'}\frac{\alpha_h^R}{6(n_{E'}+2)}-\sum_{j\in D'}\frac{\alpha_j^R}{6(n_{E'}+n_j+2)}.$$

The following corollary follows directly:

Corollary 2 *Suppose $\sigma=\{E,D,X\}$ and $\sigma'=\{E',D',X'\}$ are stable configurations with $X=X'$. Then:*

1. If $E'\subset E$ then σ Pareto dominates σ'.
2. If $n_g=n_j$ and $\alpha_g=\alpha_j$ for all g,j then σ Pareto dominates σ' if and only if $n_E>n_{E'}$.
3. If $n_g=n_j$ for all g,j and if $n_E>n_{E'}$ then σ Pareto dominates σ'.
4. If $n_g=n_j$ for all g,j and $n_E=n_{E'}$ then σ Pareto dominates σ' if and only if $\sum_{g\in E}\alpha_g<\sum_{j\in E'}\alpha_j$.

While stable equilibria corresponding to different associations can generally not be Pareto ranked, it is always the case that members of an association receive a higher level of expected utility by associating than by exit. If institutional factors such as policy weights α can be utilized to increase the size of a stable association, then this new equilibrium will dominate the former equilibrium with respect to something similar to Rawlsian (max-min) social welfare, in that the new association raises the expected utility of one or more groups above minimum levels. Section 5 works through an example in which institutional mechanisms can be varied in an attempt to grow an association.

3.2 Two groups

Before concluding this section, it is useful to consider the (much simplified) conditions for truthful messaging and voluntary association for the case of two groups, g and h. In this setting, truthful messaging is satisfied for group g if

$$\frac{1}{2(n_g+n_h+2)}\geq|\beta_h-\beta_g|.$$

Clearly this condition holds for group g if it also holds for group h; thus, regardless of preference divergence, regardless of the relative sizes of the two groups and regardless of the relative policy-making authority of the two groups (α), each group faces the same incentive to message truthfully and the incentive is wholly dependent on the preference divergence of the two groups and the total size of the population, $n_g + n_h$. Of course, there may be stable configurations of two groups in which one engages and one disengages; in these cases, however, since the truthful messaging condition is satisfied for both groups and both have chosen to associate, the configuration is Pareto dominated by another in which both choose to engage.

Group-level differences arise, however, when evaluating incentives to associate or exit, and the α terms come into play in this evaluation. If both groups message truthfully the voluntary association condition for group g is

$$\alpha_h(\beta_h - \beta_g)^2 \leq \frac{1}{6(n_g + 2)} - \frac{1}{6(n_g + n_h + 2)},$$

which requires that the weighted squared distance of their biases must be less than the expected variance of θ conditional on n_g signals minus the expected variance of θ conditional on $n_g + n_h$ signals. If both groups babble then the voluntary association condition cannot be met for both groups, by Corollary 1.

With two groups the truthful messaging condition will, in general, bind before the voluntary association condition. In other words, when both groups have an incentive to message truthfully to each other within an association, they nearly always prefer association (for a messaging equilibrium where both message truthfully) to exit. In particular, they always prefer association to exit when each group contains two or more individuals, or when the α terms are proportional to group population, so that $\alpha_g = \frac{n_g}{n_g+n_h}$ and $\alpha_h = \frac{n_h}{n_g+n_h}$. At the same time, if relative policymaking discretion, α, is highly disproportional to the groups' relative sizes and one group contains a single individual, then there can be instances in which mutual engagement is a messaging equilibrium but is not stable; the larger group prefers exit to association.

4 Association dynamics

The truthful messaging condition defined in Condition 1 has a clear interpretation: if the impact of a sender's lie on the activities of others is greater than (a function of) the preference divergence between the sender and the other members of its association, then that lie shifts the activities of the other groups too much. In this case, lying is not beneficial, and truthful communication from the sending group to the association can be sustained in a messaging equilibrium. In this section I consider the special case of three groups in order to better understand the equilibrium dynamics of truthful messaging, association and exit. The three group setting provides the simplest illustration of the effect of one group on the dynamics of a preexisting association (or non-association). More specifically, the setting enables us to consider some simple comparative statics: pinning down the size, policymaking discretion and locations of two groups we can study how the presence of the third group alters association and communication strategies as we vary parameters that characterize the third group.

The following examples illustrate two cases of interest; in the first, the entry of a third group breaks a preexisting association between groups 1 and 2. In the second, the third group's entry enables association between these groups where previously association was impossible.

Example 1 *A "moderately extreme" third group may hinder beneficial association.*

Suppose that groups 1 and 2 are identical with respect to size and that policymaking discretion (α) is directly proportional to the size of each group. The example uses parameter values $\beta_1 = 0; \beta_2 = .05; n_1 = n_2 = 2$ and $n_3 = 1$. At these parameter values 1 and 2 have biases that are sufficiently close to each other to enable truthful messaging within association $R = \{1, 2\}$.

Starting at $\beta_3 = 0$, Figure 1 shows a Pareto-optimal and stable equilibrium for each value of $\beta_3 \geq 0$.[10]

[10]For some values of β_3 there are multiple Pareto-optimal equilibria, but all Pareto-optimal equilibria yield identical ex-ante expected utility to the groups.

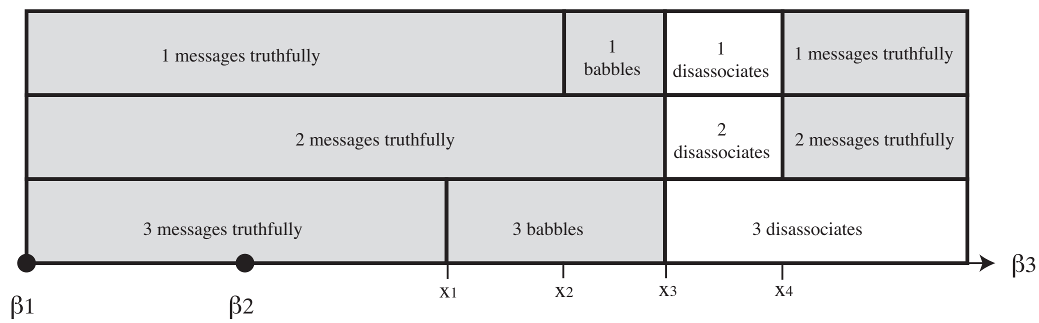

Figure 1: "Spoiler" equilibria as group 3 becomes sufficiently extreme.

- Beginning at $\beta_3 = \beta_1 = 0$, truthful messaging can be supported for all three groups until the point at which β_3 reaches x_1.

- At the point $\beta_3 = x_1 = .096$, group 3 becomes too distant from groups 1 and 2 to be able to communicate truthfully to them. Group 3 still wishes to listen to the other groups; the messages from 1 and 2 both reduce variance in its own choice and in those groups' choices. However, when group 3 babbles the value of association for all groups is reduced, as less information is being communicated.

- At $\beta_3 = x_2 = .149$ group 3 becomes too distant for group 1 to be able to communicate truthfully to an association containing group 3. At this point, both 1 and 3 remain in the association in order to listen to group 2, which is still incentivized to communicate truthfully to 1 and 3. At the same time, with fewer signals being communicated to the association, the value of association again decreases.

- At $\beta_3 = x_3 = .176$ the value of association for group 1 becomes negative, and group 1 exits the association. While truthful communication by 2 to groups 1 and 3 is a messaging equilibrium for $R = \{1, 2, 3\}$ it is not stable, as 1 wishes to exit (although 3 wishes to remain in the association). At the same time, while truthful communication between groups 1 and 2 is a messaging equilibrium for $R = \{1, 2\}$ it is not stable, as group 3 would want to join that association. When group 1 exits the association, truthful communication cannot be sustained between groups 2 and 3. Therefore there is only one stable equilibrium, and it corresponds to exit by all groups.

- Finally, at $\beta_3 = x_4 = .186$ group 3 would no longer wish to be in an association with groups 1 and 2. From this point on groups 1 and 2 can form a stable association without a threat of entry by 3.

Example 2 *A third group may induce beneficial association.*

In the previous example groups 1 and 2 were kept from communicating with each other by the presence of a third group. In the following example the third group is able to induce association between 1 and 2 where previously those groups could not associate. As before, suppose that groups 1 and 2 are identical with respect to size ($n_1 = n_2 = 2$), that $n_3 = 1$ and that the α terms are proportional to group size. However, now let $\beta_1 = 0$ and $\beta_2 = .085$; these biases are too far apart for truthful messaging to be sustained within association $R = \{1, 2\}$. Again, for each $\beta_3 \geq 0$, Figure 1 shows a Pareto-optimal and stable equilibrium.[11]

- Beginning at $\beta_3 = \beta_1 = 0$, truthful messaging can be supported by groups 1 and 3; group 2 is too distant to communicate truthfully, but associates with 1 and 3 in order to utilize their information.

- From $\beta_3 = x_1 = .040$ until $\beta_3 = x_2 = .044$, group 3 is sufficiently moderate with respect to β_1 and β_2 that truthful messaging can be sustained between all three groups.

- As β_3 moves above $x_2 = .044$ group 3 becomes too distant for group 1 to be able to communicate truthfully to an association containing groups 2 and 3. 1 remains in the association in order to listen to 2 and 3, which are sufficiently close to each other to be incentivized to communicate truthfully to association $\{1, 2, 3\}$.

[11] Again, for some values of β_3 there are multiple Pareto-optimal equilibria, but all Pareto-optimal equilibria yield identical ex-ante expected utility to the groups.

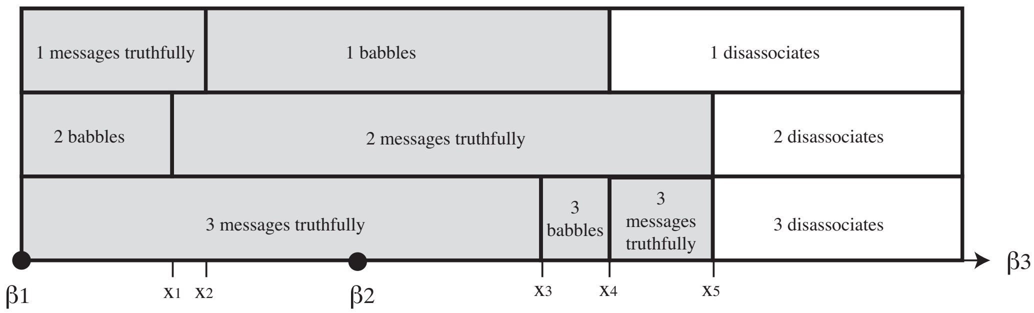

Figure 2: Group 3 induces (beneficial) universal association.

- At $\beta_3 = x_3 = .13$ group 3 has become too distant from 1 and 2 to be able to communicate truthfully to them and resorts to babbling. However 2 still communicates truthfully to 1 and 3 and the value of association remains positive for all groups.

- At $\beta_4 = .147$ the value of association for group 1 becomes negative, and group 1 exits the association. With 1's exit group 3 is now able to communicate truthfully to 2 (and vice-versa, as in the 2-group case the truthful messaging condition is identical for both groups).

- Finally, at $\beta_3 = x_5 = .185$ truthful communication can no longer be sustained between groups 2 and 3. At this point both exit and the unique stable equilibrium corresponds to exit by all groups.

5 Policy discretion as inducement

The α terms can be used to incentivize certain groups to both associate and to truthfully communicate. In the example that follows, let there be three groups with $n_1 = 2, n_2 = 4$ and $n_3 = 1$, and with $\beta_1 = 0$, $\beta_2 = .07$ and $\beta_3 = .17$. In this case group 2 is both considerably larger than groups 1 and 3, and the smallest group, group 3, is a preference outlier in the sense of being farther from median group 2 than group 1 is. In this example any association containing any pair of groups cannot sustain truthful communication; each group is simply too far from any other group and/or too well-informed. At the same time, groups 1 and 3 would benefit from association with group 2 to the extent that yielding all policy-making authority to 2 would be preferable to exit for these groups.

While group 2's truthful communication and voluntary association conditions trivially hold at $\alpha_2 = 1$, the equality is knife-edged; letting $\alpha_1 = .001, \alpha_2 = .998$ and $\alpha_3 = .001$, for example, breaks 2's incentive to associate. Moreover, there may be normative reasons to grant a positive degree of policy-making discretion to every group, in keeping with Young's argument that "...a democratic decision is normatively legitimate only if all those affected by it are included in the process of discussion and decision-making."[12] The question then is whether there exists a strictly positive vector of policy-making weights, α, for which all groups have a strictly positive incentive to associate, and for which some information can be shared across groups. In this example I assume $\sum_i \alpha_i = 1$, so that there is no artificial benefit ($\sum_i \alpha_i < 1$) or penalty ($\sum_i \alpha_i > 1$) stemming from the association decision.

To answer this question, first note that there is no strictly positive α than can ever induce either groups 1 or 3 to truthfully communicate to an association consisting of $\{1, 2, 3\}$. Truthful communication for 1 requires that

$$\frac{\alpha_2}{m_2 + 4}\left(\frac{1}{2(m_2 + 4)} - .07\right) + \frac{\alpha_3}{m_3 + 4}\left(\frac{1}{2(m_3 + 4)} - .17\right) \geq 0,$$

and truthful communication for 3 requires

$$\frac{\alpha_1}{m_1 + 3}\left(\frac{1}{2(m_1 + 3)} - .17\right) + \frac{\alpha_2}{m_2 + 3}\left(\frac{1}{2(m_2 + 3)} - .1\right) \geq 0.$$

The incentive is easiest to satisfy for each group when the m_i (number of signals received by group i) are minimized, which occurs at $m_i = n_i$; both inequalities fail at this point. It follows that the only possible association consisting of all three groups would correspond to $\{2\} = E$ and $\{1, 3\} = D$.

If voluntary association is nontrivially satisfied for 2 at some α then, by Corollary 1, it must be the case that 2 is incentivized to communicate truthfully to 1 and 3. If this is the case, then 2's

[12] Young (2002, p. 23).

association condition becomes:

$$-\alpha_1(.07)^2 - \alpha_3(.1)^2 - \alpha_1(\frac{1}{48}) - \alpha_3(\frac{1}{42}) - (1 - \alpha_1 - \alpha_3)(\frac{1}{36}) + \frac{1}{36} \geq 0.$$

This equation reduces to the requirement that $\alpha_1 \geq 2.95\alpha_3$. However, satisfaction of this association requirement does not imply that $\{\{2\}, \{1, 3\}\}$ is a messaging equilibrium for association $\{1, 2, 3\}$. To see this, note that when $\alpha_3 = 0$ the above equation is satisfied for 2 but, as 2's message only has a tangible effect on 1's action, truthful communication cannot be sustained. In this case 2's truthful messaging condition is identical to its messaging condition within an association consisting of only 1 and 2 and, by design, the condition is not satisfied for the pair. Group 2 would want to associate with groups 1 and 3 if 2 could credibly communicate to them, but 2's message is not credible. Satisfaction of truthful communication for 2 additionally requires that

$$\frac{\alpha_1}{128} + \frac{\alpha_3}{98} - \left|\frac{.07\alpha_1}{8} - \frac{.1\alpha_3}{7}\right| \geq 0.$$

This condition reduces to the pair of inequalities $.25\alpha_3 \leq \alpha_1 \leq 26.12\alpha_3$.

To summarize, there *does* exist a collection of strictly positive policy-making weights α for which the association $E = \{2\}, D = \{1, 3\}$ is stable, and for which each group's benefit to association is strictly positive. Letting Δ^2_+ be the interior of the 2-dimensional unit simplex, this set is defined as $\{(\alpha_1, \alpha_2, \alpha_3) \in \Delta^2_+ : 2.95\alpha_3 \leq \alpha_1 \leq 26.12\alpha_3\}$.

To see how varying the distribution of α across the three groups affects group 2's association and messaging conditions I present two figures. In Figure 3 I explicitly plot out the net benefits of truthful messaging and association for group 2 as a "proportionality index" P is varied. For a given association R, let p_i be the proportion of the population in group i, or $p_i = \frac{n_i}{\sum_{j \in R} n_j}$. P then generates α as follows: $\alpha_i(P) = \frac{p_i^P}{\sum_{j \in R} p_j^P}$. Thus, at $P = 0$ policy-making authority is equally distributed across the groups, irrespective of size; in this example $P = 0$ corresponds to $\alpha(0) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. At $P = 1$ authority is directly proportional to group size, so that $\alpha(1) = (\frac{2}{7}, \frac{4}{7}, \frac{1}{7})$. As $P \to \infty$, α approaches $(0, 1, 0)$.

Figure 3 shows that for low values of P truthful messaging can be sustained for 2, but voluntary association cannot; the reason for this is that when 3 is given too much authority relative to 1,

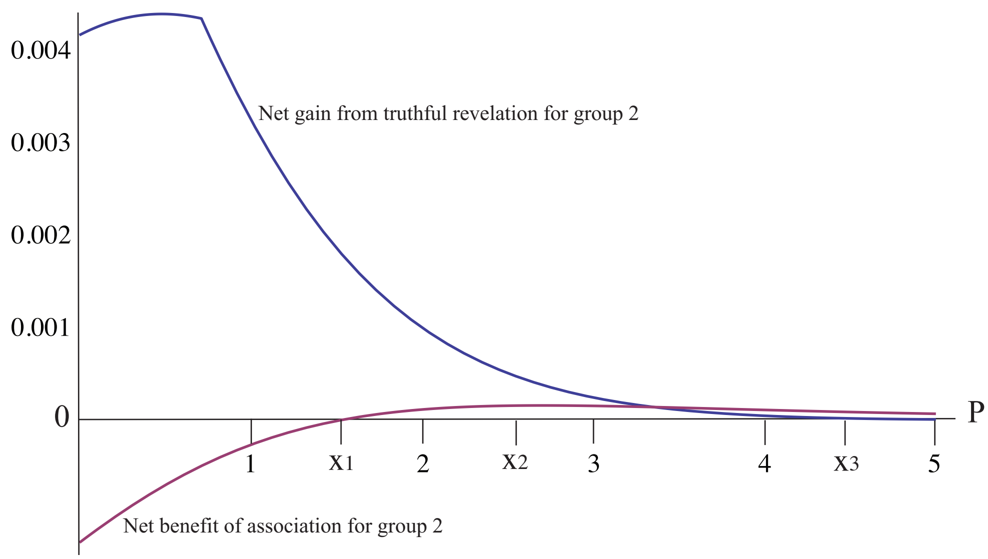

Figure 3: Varying an index of proportionality changes 2's incentives.

2's utility decreases in two ways; first, 2 suffers greater disutility from 3's more extreme bias relative to 1, but second, 2 also loses the benefit of 1's extra signal relative to 3. Since 1 and 3 cannot message truthfully, the value of their information is only realized through their own activity choices. Thus, 2 benefits doubly when 1 is granted greater authority than 3. In Figure 3 the bottom curve represents 2's net gain from association versus exit while the top curve represents 2's net gain from truthful communication. The figure shows that 2 is incentivized to associate only when P exceeds $X_1 = 1.56$, which corresponds to $\alpha(1.56) = (.23, .69, .08)$. 2 receives its maximum benefit from association when P reaches $X_2 = 2.7$, corresponding to $\alpha(2.7) = (.13, .85, .02)$. As noted earlier, when 2 is granted all policy-making authority it is indifferent between association and exit. The fact that the value of association for 2 is maximized at an interior point reflects the fact that 2 can strictly benefit from the decisions made by 1 and 3 when those groups are given the additional information that 2 possesses.

When P reaches $x_3 = 4.7$, group 2's truthful messaging condition fails to hold; at this point group 1 is receiving too much authority relative to 3 for 2 to be incentivized to be truthful. Prior to this point, 2's incentive to manipulate 1's choice was kept in check by the fact that such manipula-

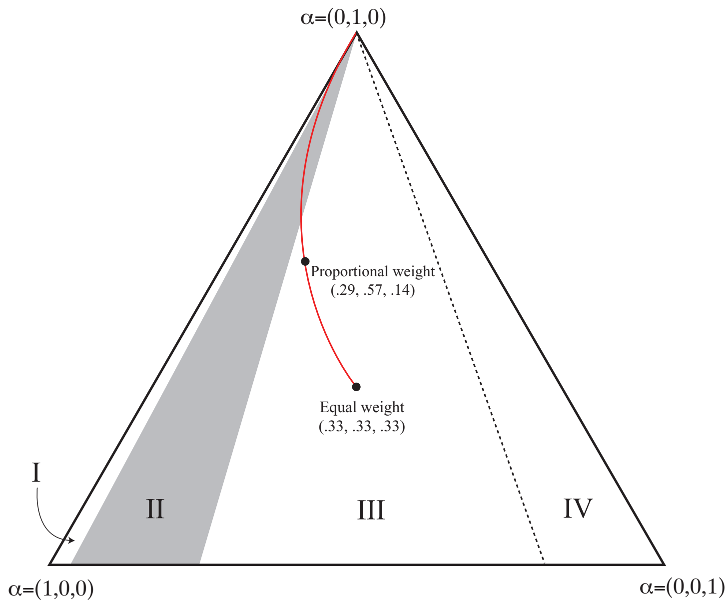

Figure 4: 2's incentives for all possible distributions of α.

tion would also generate a costly change in 3's choice. As 3's authority becomes too low relative to 1's, 3's presence can no longer prevent 2 from benefiting from manipulating 1. This point approximately corresponds to $\alpha(4.7) = (.037, .962, .001)$. It follows from the inequality derived above requiring that $\alpha_1 \leq 26.12\alpha_3$ be satisfied for truthful messaging to hold for 2.

Figure 4 depicts the Pareto-optimal, stable equilibria for all possible realizations of α. The vertices of the simplex represent the values of α at which one player is granted all policy-making authority, with the top of the simplex representing the point at which 2 has all authority. The collection of α's characterized by proportionality index $P \geq 0$ is represented by the red curve. Although difficult to see in the figure, this curve leaves region II as it approaches $\alpha = (0, 1, 0)$. The shaded region II represents the values of α for which 2 truthfully messages to 1 and 3, and for which the association decision is positive for all groups. In region I group 2 cannot be compelled to truthfully communicate to 1 and 3, because (as described above) α_3 is too low relative to α_1 for group 3 to effectively counterbalance group 1's bias. In region III group 2 *can* truthfully communicate with 1 and 3, but prefers exit to association. Again, as described above, in this region α_3 is too large relative to α_1 for association to be beneficial to 2, because for large α_3's 2 must endure 3's greater bias and lower information. In region IV, group 2 no longer wants to communicate with 1 and 3, because α_1 is too low relative to α_3 for 1 to counterbalance 3's bias.

Interestingly, the absolute levels of α_1 and α_3 don't affect 2's decision to either associate or communicate truthfully; only the relative levels of these terms matter. 2 can be incentivized to communicate truthfully and strictly prefer association to exit even when it is granted zero policy-making authority. It is also important to note that truthful messaging by 2 and voluntary association by all groups can only be meaningfully sustained when both $\alpha_1 > 0$ and $\alpha_3 > 0$. Significantly, even though group 3 is a preference outlier and possesses little information of its own, its nontrivial presence in the association is necessary in order to induce 2 to communicate truthfully.

6 Exclusion versus obfuscation

If the truthful messaging condition fails for a group it is not difficult to find cases in which the group foregoes some of its own information in order to make its messages more credible to an outgroup, and in which giving up this information is a Pareto improvement. Consider a two group example in which $n_1 = n_2 = 7$, $\beta_1 = 0, \beta_2 = .033$ and $\alpha_1 = \alpha_2 = .5$. Truthful communication requires $\beta_2 \leq .03125$ and so the only stable configuration of groups is one in which both 1 and 2 exit and receive expected payoffs of $-.0185$. However, if 2 commits to giving up a signal–essentially ignoring some of its own information–messaging can be sustained for $\beta_2 \leq 0.03333$. The payoff to association in this case is $-.01165$, and so 2 is made strictly better off: by giving up one of its own signals 2 is able to gain the 7 signals of group 1.

At the same time, this comparison is unsatisfying because there may exist other messaging equilibria that Pareto dominate the equilibrium 2 generates by foregoing a signal. Up until now I have focused exclusively on fully separating equilibria in which a group sending a message to an association must be incentivized to be truthful for any number of positive signals it has received; if the group ever wishes to misrepresent its true number of positive signals, all other groups receiving its message assume the message is uninformative. The focus on separating equilibria clearly simplifies the analysis, and, importantly, the substance of the results presented so far would not change if I expanded my scope to consider semi-separating equilibria. The dynamics in the examples I have presented stem from the tension between the "congestion effect" of more signals (more signals make the effect of a lie smaller, and thus reduce incentives for truthful messaging), and from the "association effect" of more signals (more signals reduce variance, and thus increase incentives for association). Both effects will also be present if considering semi-separating equilibria.

It is not clear, however, that foregoing one's own information is beneficial if groups play semi-separating equilibria. In the remainder of this section I derive the best partitional equilibrium for the parameters given above, and show that it marginally outperforms the separating equilibrium in which group 2 gives up a signal.

Let $\beta_1 = 0, \beta_2 > 0$ and $n_1 = n_2 = 7$. As described in Section 2.2, a (pure) strategy for each group is a mapping $\rho_g : \{0, 1, ..., n_g\} \times 2^{|G|} \to \{0, 1, 2, ..., n_g\}$ from the set of types g could be, and the set of groups in its association, into a public message, m_g. If group g plays a semi-separating strategy then the range of ρ_g has strictly more than one element in it and strictly less than $n_g + 1$: some types that g could be are mapped into the same message.

Suppose group 1 observes a number of positive signals s_1 and 2's message m_2. Then by sequential rationality 1 chooses y_1^* by calculating the probability that 2 is each of the types mapped into m_2. At the same time, 2's incentive to deviate from messaging strategy m_2 is calculated by computing the expected value of y_1^* conditional on 2 sending correct message m_2 versus some different m_2', and conditional on 2 having received s_2 positive signals.

Recall that if m total signals are received, the probability that $k = t$ are positive is

$$Pr[k = t|m] = \int_{\theta=0}^{1} \binom{m}{t} \theta^t (1-\theta)^{m-t} d\theta = \frac{1}{m+1}.$$

If m total signals are received and k are positive, the posterior distribution of θ is

$$f(\theta|k, m) = (m+1)\binom{m}{k}\theta^k(1-\theta)^{m-k}.$$

Therefore if 1 has received n_1 signals and s_1 signals were positive, its posterior belief that 2 has received $s_2 = k$ positive signals is:

$$Pr[s_2 = k|s_1] = \int_{\theta=0}^{1} \binom{n_2}{k} \theta^k (1-\theta)^{n_2-k} f(\theta|s_1, n_1) d\theta.$$

These posterior beliefs for $n_1 = n_2 = 7$ are shown in the table below, with the columns representing the positive signals one group has observed and the rows representing the likelihood the other group received that number of positive signals.

	Observed positives							
Posterior	0	1	2	3	4	5	6	7
0	$\frac{8}{15}$	$\frac{4}{15}$	$\frac{8}{65}$	$\frac{2}{39}$	$\frac{8}{429}$	$\frac{4}{715}$	$\frac{8}{6435}$	$\frac{1}{6435}$
1	$\frac{4}{15}$	$\frac{56}{195}$	$\frac{14}{65}$	$\frac{56}{429}$	$\frac{28}{429}$	$\frac{56}{2145}$	$\frac{49}{6435}$	$\frac{8}{6435}$
2	$\frac{8}{65}$	$\frac{14}{65}$	$\frac{168}{715}$	$\frac{28}{143}$	$\frac{56}{429}$	$\frac{49}{715}$	$\frac{56}{2145}$	$\frac{4}{715}$
3	$\frac{2}{39}$	$\frac{56}{429}$	$\frac{28}{143}$	$\frac{280}{1287}$	$\frac{245}{1287}$	$\frac{56}{429}$	$\frac{28}{429}$	$\frac{8}{429}$
4	$\frac{8}{429}$	$\frac{28}{429}$	$\frac{56}{429}$	$\frac{245}{1287}$	$\frac{280}{1287}$	$\frac{28}{143}$	$\frac{56}{429}$	$\frac{2}{39}$
5	$\frac{4}{715}$	$\frac{56}{2145}$	$\frac{49}{715}$	$\frac{56}{429}$	$\frac{28}{143}$	$\frac{168}{715}$	$\frac{14}{65}$	$\frac{8}{65}$
6	$\frac{8}{6435}$	$\frac{49}{6435}$	$\frac{56}{2145}$	$\frac{28}{429}$	$\frac{56}{429}$	$\frac{14}{65}$	$\frac{56}{195}$	$\frac{4}{15}$
7	$\frac{1}{6435}$	$\frac{8}{6435}$	$\frac{4}{715}$	$\frac{8}{429}$	$\frac{2}{39}$	$\frac{8}{65}$	$\frac{4}{15}$	$\frac{8}{15}$

Since truthful messaging is not satisfied for either group in this example, an asymmetric semi-separating equilibrium where one group is truthful and the other is partially informative does not exist; the condition will be broken for the truthful group. Therefore any semi-separating equilibrium involves a non-trivial bundling of types for both groups. Moreover, the bundling cannot involve two consecutive types being in singleton sets, because this offers a player the ability to misrepresent a single signal; in this setting the incentive to misrepresent a single signal is independent of the number of signals actually received. An example (which we can ultimately show is the best partitional equilibrium) is the following mapping:[13]

$$\rho_2(0) = \rho_2(1) = m_2^1; \rho_2(2) = m_2^2; \rho_2(3) = \rho_2(4) = m_2^3; \rho_2(5) = m_2^4; \rho_2(6) = \rho_2(7) = m_2^5. \quad (7)$$

Given that 2 is playing ρ_2, 1 calculates $y_1^*(m_2|s_1)$ as follows:

$$y_1^*(m_2|s_1) = \sum_{k:\rho(k)=m_2} \frac{Pr[s_2 = k|s_1] * E(\theta|k + s_1, n_1 + n_2)}{\sum_{j:\rho(j)=m_2} Pr[s_2 = j|s_1]}. \quad (8)$$

Thus, conditional on each type 2 could be and message 2 could send if playing ρ_2, 2 calculates the expected value of $y_1^*(m_2)$ as

$$E(y_1^*(m_2)|s_2) = \sum_{k=0}^{n_1} y_1^*(m_2|s_1) * Pr[s_1 = k|s_2].$$

Using the table above to calculate $Pr[s_g = k|s_h]$, 2's truthful messaging conditions are calculated for each s_2 and m_2 by evaluating the maximum difference in bias that that the two groups could have for which 2 has no incentive to deviate from ρ_2. For the example of ρ_2 described in Equation 7, truthful messaging holds for 2 for $\beta_2 \leq .0408$. When $\beta_2 = .033$, letting $\rho_1 = \rho_2$ as described in Equation 7 can be shown to represent the best partitional equilibrium for both groups. Letting $y_2^*(m_1|s_2)$ be calculated analogously to y_1^* in Equation 8, the expected utility each group is expected to receive is calculated as:

$$EU_1(y^*|\rho_1, \rho_2) = \sum_{k=0}^{n_1}\sum_{j=0}^{n_2} \frac{Pr[s_2 = j|s_1 = k]}{n_1 + 1} \int_{\theta=0}^{1} \left(-.5(y_1^*(m_2|s_1) - \theta)^2 - .5(y_2^*(m_1|s_2) - \theta)^2\right) f(\theta|k + j, n_1 + n_2)d\theta$$

and

$$EU_2(y^*|\rho_1, \rho_2) = \sum_{k=0}^{n_1}\sum_{j=0}^{n_2} \frac{Pr[s_2 = j|s_1 = k]}{n_1 + 1} \int_{\theta=0}^{1} \left(-.5(y_1^*(m_2|s_1) - \theta - \beta_2)^2 - .5(y_2^*(m_1|s_2) - \theta - \beta_2)^2\right) f(\theta|k+j, n_1+n_2)d\theta.$$

By the symmetry of the problem these values are identical, and equal $-.011634$.

Returning to our original question of whether the semi-separating strategy represents an improvement for 2 relative to foregoing a signal: in this case it does. The groups received an expected

[13] The mapping represents a slight abuse of notation as it disregards the R element of the domain of ρ_2; since there are only two groups (and thus one possible association) it is unnecessary.

utility of $-.01165$ when 2 gave up a signal, but receive $-.011634$ when playing the best partitional strategy. Whether welfare could be improved in this game by a reduction in information remains an open question when considering the possibility of semi-separating equilibria; in this example the net payoff to the partitional equilibrium relative to the equilibrium corresponding to a lost signal is small. If the answer is "yes" it will stem from the fact that while a partially informative messaging strategy can always be used to conceal information from a receiving group, that strategy cannot prevent the sending group from observing all of its own signals, and thus cannot mitigate the congestion effect stemming from this private information.

7 Limiting within-group communication

The previous section explored one possible intra-group mechanism for improving inter-group communication: in the event that truthful communication cannot be sustained across groups, it can potentially be beneficial for a group to "give up a signal" in order to reduce the congestion effect of too much information and to enable communication. At the same time, that section showed that foregoing a signal in this way did not represent an improvement over the group partially revealing its information through a semi-separating strategy. In this section I consider a different intra-group mechanism for improving communication across groups: limiting intra-group communication prior to the public messaging stage. Consequently I set aside the conception of the group as a unitary actor for the remainder of this section, and this will be shown to be in the group's interest.

Suppose there are two groups, 1 and 2, with $\beta_1 = 0$ and $\beta_2 > 0$. Truthful messaging for these groups requires that

$$\frac{1}{2(n_1 + n_2 + 2)} \geq \beta_2.$$

Now suppose that each group prohibits intra-group communication prior to the public messaging stage. In this case, each person approaches the association with only their own signal to reveal. Last, suppose that policy-making authority has been disaggregated within each group, so that the activity choice of each $i \in g$ receives weight α_{gi}, with $\sum_i \alpha_{gi} = \alpha_g$ and with $\alpha_1 + \alpha_2 = 1$.

The condition for truthful messaging by an individual $i \in g$ to the association consisting of all members of both groups is now

$$\sum_{j\in h} \frac{\alpha_{hj}}{2(n_1+n_2+2)^2} + \sum_{j\in g\setminus\{i\}} \frac{\alpha_{gj}}{2(n_1+n_2+2)^2} \geq \sum_{j\in h} \frac{\alpha_{hj}}{(n_1+n_2+2)}\beta_2$$

which reduces to

$$\frac{1}{2(n_1+n_2+2)^2} - \frac{\alpha_{gi}}{2(n_1+n_2+2)^2} - \frac{\alpha_h}{(n_1+n_2+2)}\beta_2 \geq 0. \tag{9}$$

Equation 9 shows that when information is disaggregated within each group, truthful messaging is easier to achieve; group members are more incentivized to be truthful because a false message not only biases the choices of members of the outgroup, but also biases members of their own group in an undesirable way. This is akin to making the group's signal costly, and bears some similarity to the models developed in Patty and Penn (2012) and Gailmard and Patty (2013). Equation 9 also demonstrates that there is an optimum distribution of authority, both across groups and within groups, that maximizes the incentive for universal communication. Since the messaging condition binds more strongly as α_{gi} and α_h increase, the condition is easiest to satisfy when the maximum value these terms could take is minimized, which occurs at $\alpha_1 = \alpha_2 = \frac{1}{2}$, and $\alpha_{gi} = \alpha_{gj} = \frac{\alpha_g}{n_g}$. Thus, an even distribution of power maximizes incentives for universal communication.

To show that limiting intra-group communication in this way is particularly beneficial, I return to the example of the best semi-separating equilibrium presented in Section 6. In the example, $n_g = n_h = 7$, $\beta_2 = .033$ and $\alpha_1 = \alpha_2 = \frac{1}{2}$. The example showed that in the optimal semi-separating messaging equilibrium signals are mapped into five messages. The expected utility to each player from association is $-.011649$. In the example truthful messaging (a separating messaging equilibrium) cannot be sustained for $\beta > 0.03125$. However, if intra-group communication is prohibited prior to the public message, Equation 9 reveals that truthful messaging for all individuals can be sustained for $\beta < .058$ (when $\alpha_i = \frac{1}{14}$ for each i). Moreover, since all information is transmitted to all groups, the equilibrium with disaggregated information is payoff equivalent to truthful messaging between groups 1 and 2 (which could not be sustained in a separating equilibrium with aggregated information). This equilibrium necessarily Pareto dominates the best semi-separating equilibrium, as it involves more information transmission. In particular, it yields each member of the association an expected utility of $-.01096$.

8 Conclusions

In this paper I considered the possibility of information transmission across groups in a setting in which groups hold different biases and different amounts of information, any group may choose to unilaterally exit society, and communication is costless. Focusing on separating equilibria, the paper characterizes the "associations of groups" that can be sustained in equilibrium and, within those associations, the types of groups that choose to engage in or disengage from the process of inter-group communication. These associations are the collections of groups for which some information transmission between groups is possible, and for which the benefits of this information outweigh the costs stemming from preference diversity within the association.

Several insights emerge from the model. First, the presence of a third group may induce association between two groups that could not associate previously, even if the third group's bias is more extreme than the biases of the other two. At the same time, situations can also arise in which the presence of a third group can extinguish any possibility of communication between groups, where previously communication was possible. Second, institutional mechanisms governing the amount of policy discretion held by each group can be used as a lever to induce beneficial communication and universal association, even in settings in which communication is impossible between any pair of groups. These mechanisms may be highly disproportional, and may need to be calibrated to ensure satisfaction of both the association and messaging conditions for the group(s) the institutional designer wishes to induce to "talk." And third, certain mechanisms may be used *within* a group in order to make that group's message more credible to an association. These mechanisms include excluding some information from a group's own pool of information (by, for example, refusing to acknowledge a group member's signal); obfuscating information through the use of a semi-separating strategy; and limiting within-group communication prior to the public messaging stage, so that information transmitted to an outgroup must also be transmitted to the ingroup (thus rendering a lie more costly). Many topics for future research remain.

9 Appendix

Proof of Lemma 1. If group g has n_g members then it can state that its members have received any number of signals in $\{0, ..., n_g\}$. We start by supposing that revealing some $\tilde{s} > s_g + 1$ represents a profitable lie for g to make, in the sense of biasing the information of the other groups in a way that is attractive to g. We will show that if this is the case, then it must be the case that revealing $s^o = s_g + 1$ is also a profitable lie for g.

For each $j \in R \setminus \{g\}$, let k_j be the sum of truthful positive signals received by j (excluding group g's information) and m_j be the total number of truthful signals observed by j, again excluding g's information. If it is profitable for g to lie with $\tilde{s}$ it must be that:

$$-\sum_{j\in R\setminus\{g\}} \alpha_j^R \left(\beta_j + E(\theta|k_j + \tilde{s}, m_j + n_g) - \beta_g - E(\theta|k_j + s_g, m_j + n_g)\right)^2 + \sum_{j\in R\setminus\{g\}} \alpha_j^R \left(\beta_j - \beta_g\right)^2 > 0.$$

Expanding and collecting terms, this implies:

$$\sum_{j\in R\setminus\{g\}} \alpha_j^R \left(E(\theta|k_j + \tilde{s}, m_j + n_g) - E(\theta|k_j + s_g, m_j + n_g)\right)\left(2\beta_j - 2\beta_g + E(\theta|k_j + \tilde{s}, m_j + n_g) - E(\theta|k_j + s_g, m_j + n_g)\right) < 0.$$

Plugging in the values for $E(\theta|\cdot)$, we get that:

$$\sum_{j\in R\setminus\{g\}} \alpha_j^R \left(\frac{\tilde{s} - s_g}{m_j + n_g + 2}\right)\left(2\beta_j - 2\beta_g + \frac{\tilde{s} - s_g}{m_j + n_g + 2}\right) < 0,$$

and dividing both sides by $\tilde{s} - s_g$, which we can do because $\tilde{s} - s_g > 0$, we get:

$$\sum_{j\in R\setminus\{g\}} \left(\frac{\alpha_j^R}{m_j + n_g + 2}\right)\left(2\beta_j - 2\beta_g + \frac{\tilde{s} - s_g}{m_j + n_g + 2}\right) < 0.$$

Since we assumed that $\tilde{s} > s_g$, it follows that the term $\frac{\tilde{s} - s_g}{m_j + n_g + 2} > 0$ for all j. This implies that if the above condition holds for some $\tilde{s} > s_g + 1$, then it also holds for $s_g + 1$. A similar argument holds if we had instead assumed that $\tilde{s} < s_g - 1$ represented a profitable deviation for g. I omit this half of the proof. □

Proof of Condition 1. The proof is similar to that of Galeotti et al. (2013). Let $E_{-g} = E \setminus \{g\}$, $D_{-g} = D \setminus \{g\}$. In the same fashion, let $s_{E_{-g}} = \sum_{h \in E_{-g}} s_h$, the sum of positive signals received by the groups in E_{-g}, and let $n_{E_{-g}}$ be the total number of signals received by the groups in E_{-g}. Let R be an association, with $h \in E \subseteq R$ being the groups that truthfully reveal their signals and $j \in D \subseteq R$ being those that babble. As shown in Lemma 1, if g is incentivized to communicate truthfully, then g receives no benefit from misrepresenting the signal of a single one of its members. For a group $h \in E$ the total observed number of (true) positive signals is s_E and for a group $j \in D$, the total observed number of true positive signals is $s_E + s_j$, as j observes the signals of individuals in all groups that have chosen to engage, and also the private signals of its own members.

Group g's expected payoff to truthful communication to an association consisting of itself and $\{E_{-g}, D_{-g}\}$ is

$$
\begin{aligned}
&- \sum_{h \in E_{-g}} \left[\alpha_h^R \left(y_h^*(s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - E(\theta | s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - \beta_g \right)^2 \right] \\
&- \sum_{j \in D} \left[\alpha_j^R \left(y_j^*(s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - E(\theta | s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - \beta_g \right)^2 \right]
\end{aligned}
\tag{10}
$$

and group $g's$ payoff to misrepresenting the signal of a single one of its members is

$$
\begin{aligned}
&- \sum_{h \in E_{-g}} \left[\alpha_h^R \left(y_h^*(s_{E_{-g}} + s_g \pm 1, n_{E_{-g}} + n_g) - E(\theta | s_{E_{-g}} + s_g, n_{E_{-g}} + n_g) - \beta_g \right)^2 \right] \\
&- \sum_{j \in D} \left[\alpha_j^R \left(y_j^*(s_{E_{-g}} + s_g + s_j \pm 1, n_{E_{-g}} + n_g + n_j) - E(\theta | s_{E_{-g}} + s_g + s_j, n_{E_{-g}} + n_g + n_j) - \beta_g \right)^2 \right],
\end{aligned}
\tag{11}
$$

where the $\pm$ term represents whether g misrepresented a negative signal as positive or vice versa. Therefore g has an incentive to truthfully report the positive signals of its members if and only if Equation 10 $\geq$ Equation 11.

To simplify notation, for the remainder of the proof I will let $\mathbf{p_j}$ be the collection of truthful positive signals observed by group j along with g's (true) positive signals and $\mathbf{n_j}$ be the total number of truthful signals observed by group j along with g's signals. Thus, for $h \in E_{-g}$ we have $\mathbf{p_h} = s_{E_{-h}} + s_g$ and $\mathbf{n_h} = n_{E_{-g}} + n_g$. For $j \in D_{-h}$ we have $\mathbf{p_j} = s_{E_{-g}} + s_j + s_g$, and so on. Reducing, and using the fact that $(a+b)(a-b) = a^2 - b^2$, the necessary and sufficient condition for truthful communication by group g to association R is

$$-\sum_{i \in R \setminus \{g\}} \alpha_i^R (y_i^*(\mathbf{p_i}, \mathbf{n_i}) - y_i^*(\mathbf{p_i} \pm 1, \mathbf{n_i}))(y_i(\mathbf{p_i}, \mathbf{n_i}) + y_i(\mathbf{p_i} \pm 1, \mathbf{n_i}) - 2E(\theta|\mathbf{p_i}, \mathbf{n_i}) - 2\beta_g) \geq 0.$$

Dividing both sides by 2 and letting y_i^* be defined as in Equation 5, we get

$$-\sum_{i \in R \setminus \{g\}} \alpha_i^R \left(\beta_i + \frac{\mathbf{p_i}+1}{\mathbf{n_i}+2} - \beta_i - \frac{\mathbf{p_i}+1 \pm 1}{\mathbf{n_i}+2}\right) \left(\frac{2\beta_i + \frac{\mathbf{p_i}+1}{\mathbf{n_i}+2} + \frac{\mathbf{p_i}+1\pm 1}{\mathbf{n_i}+2}}{2} - \frac{\mathbf{p_i}+1}{\mathbf{n_i}+2} - \beta_g\right) \geq 0.$$

Further reducing and letting $\tilde{s}_g$ be the single signal that g potentially wishes to lie about we get

$$\begin{array}{ll} -\sum_{i \in R \setminus \{g\}} \alpha_i^R (\frac{-1}{\mathbf{n_i}+2})(\frac{1}{2(\mathbf{n_i}+2)} + \beta_i - \beta_g) \geq 0 & \text{if} \quad \tilde{s}_g = 0 \\ -\sum_{i \in R \setminus \{g\}} \alpha_i^R (\frac{1}{\mathbf{n_i}+2})(\frac{-1}{2(\mathbf{n_i}+2)} + \beta_i - \beta_g) \geq 0 & \text{if} \quad \tilde{s}_g = 1, \end{array}$$

which can be rewritten as

$$\begin{array}{rcll} \sum_{i \in R \setminus \{g\}} \alpha_i^R (\frac{1}{\mathbf{n_i}+2})(\frac{1}{2(\mathbf{n_i}+2)}) & \geq & -\sum_{i \in R \setminus \{g\}} \alpha_i^R (\frac{1}{\mathbf{n_i}+2})(\beta_i - \beta_g) & \text{if} \quad \tilde{s}_g = 0 \\ \sum_{i \in R \setminus \{g\}} \alpha_i (\frac{1}{\mathbf{n_i}+2})(\frac{1}{2(\mathbf{n_i}+2)}) & \geq & \sum_{i \in R \setminus \{g\}} \alpha_i^R (\frac{1}{\mathbf{n_i}+2})(\beta_i - \beta_g) & \text{if} \quad \tilde{s}_g = 1. \end{array}$$

Last, the above can be combined into a single inequality:

$$\sum_{i \in R \setminus \{g\}} \frac{\alpha_i^R}{2(\mathbf{n_i}+2)^2} \geq \Big| \sum_{i \in R \setminus \{g\}} \frac{\alpha_i^R}{\mathbf{n_i}+2}(\beta_i - \beta_g) \Big|.$$

Substituting in the values for $\mathbf{p_i}$ and $\mathbf{n_i}$ for $j \in D_{-g}$ and $h \in E_{-g}$ yields the statement of this condition. □

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